Optimal asymmetric taxation in a two-sector model with population ageing

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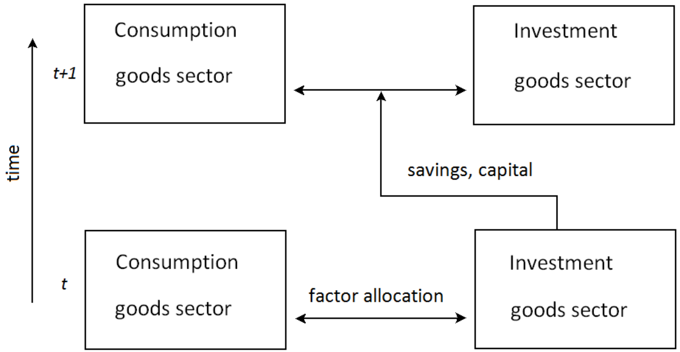
The model:

- Dynamic framework (2-period overlapping generations).
- 2-sector model:
  - consumption goods, which can be consumed, and cannot be invested;
  - investment goods, which can be invested, but cannot be consumed.

Cremers (2006):

- Agents make savings not taking into account that their savings now determine capital-labour ratios in future (Externality).
- As a result, their savings are suboptimal.

Figure: General structure of the model



Production functions

Production functions in two sectors:

$$Y_C(t) = K_C^{\alpha}(t) L_C^{1-\alpha}(t), \tag{1}$$
$$Y_I(t) = K_I^{\beta}(t) L_I^{1-\beta}(t). \tag{2}$$

- Y- Output, K - capital, L -labour.
- C, I - indexes for consumption and investment good sectors.
- Wages and interest rates are equal to the marginal products and expressed in consumption goods.

Households

Log-linear utility function:

$$U_x(t) = \log C_x^y(t) + \frac{\psi}{1+\rho} \log C_x^o(t+1), \quad x \in \{C, I\}, \qquad (3)$$

- C_x^y - Consumption when young;
- C_x^o - Consumption when old;
- ρ - Discount rate;
- ψ - Longevity (probability of surviaval before the second period of life);
- Agents work when young, get wages, consume and invest;
- Agents consume when old from savings.

Budget constraints:

$$C_x^y(t) = w_x(t)(1-\tau_x) - s_x(t), \tag{4}$$

$$C_x^o(t+1) = (1+R_x(t+1))s_x(t), \quad x \in \{C, I\}. \tag{5}$$

- s_x - Savings;
- R_x - Interest rate adjusted for annuities;
- w_x - Wage;
- τ_x -Sector specific taxes/subsidies.

FOC of utility maximization:

$$\Longrightarrow s_x(t) = \frac{\psi w_x(t)(1-\tau_x)}{1+\rho+\psi}, \quad x \in \{C, I\}. \tag{6}$$

Equilibrium

- Capital market equilibrium: returns to capital are equal in sectors I and C, $R_I = R_C$.
- Labour market equilibrium: $w_C(1-\tau_C) = w_I(1-\tau_I)$.
- Investment goods clearing condition: $K_C(t) + K_I(t) = Y_I(t-1) =$ total savings. (Capital depreciates in one period)
- Consumption goods clearing condition: $Y_C =$ total consumption.

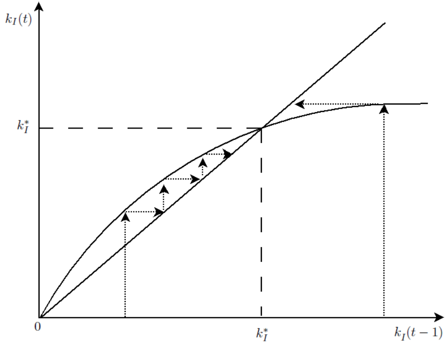
Dynamics

$$k_I(t) = \frac{k_I^{\beta}(t-1)}{1+n} \times$$
$$\left[1 + \frac{\alpha}{\beta(1-\alpha)(1-\tau_C)}\left(\frac{1+\rho+\psi}{\psi} - (1-\beta)(1-\tau_I)\right)\right]^{-1}. \quad (7)$$

- $k_I(t)$ - Capital-labour ratio in the investment good sector.

Dynamics

Figure: $k_I(t)$ dynamics



Government budget

- Government's task: To choose τ_C and τ_I, which maximize agents' utilities (for labour or capital taxes equivalent to Y_C maximization);
- Budget constraint: $L_C w_C \tau_C + L_I w_I \tau_I = G = 0.$

$$\tau_C = -\frac{\tau_I \psi (1-\beta)}{1+\rho+\beta\psi}. \tag{8}$$

Optimal labour taxation

- Optimal tax in sector I:

$$\tau_I^* = \frac{1-\alpha-\beta-\alpha(1+\rho)/\psi}{1-\beta}. \tag{9}$$

- Alternative golden rule:

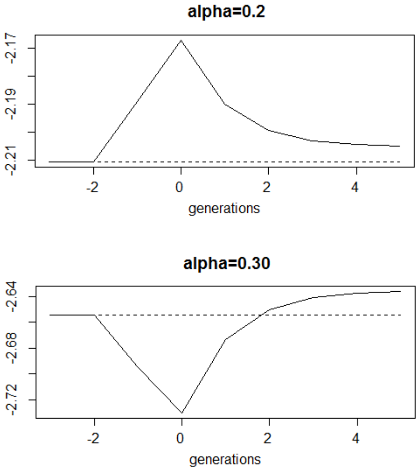
$$r^* = \frac{n(\alpha\rho+\beta\psi+\alpha)}{\psi(1-\alpha)}. \tag{10}$$

- (Standard golden rule: $r = n$).
- n - population growth in terms of fertility,
- r - interest rate unadjusted for annuities.

Numerical example

- Parameter values widely used in the literature $\beta = 0.4$, $\rho = 0.4166$, $\psi = 0.9$.

Figure: Utility



Conclusions

- Asymmetric taxation may be optimal;
- Optimality condition differs from the standard models.